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Solution of substance abuse and domestic violence mathematical model using Homotopy pertubation method

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Abstract

In this study, homotopy perturbation method was applied for the approximate solution of a mathematical model of substance abuse and domestic violence. The system of model equations developed was transformed into series of differential equations which enables the series to converge which gives the approximate solution required.

Keywords: substance abuse; domestic violence; homotopy pertubation method (hpm)

1 Introduction

Domestic Violence the violence occurring in the family or domestic unit, including, inter alia, physical and mental aggression, emotional and psychological abuse, rape and sexual abuse, incest, marital rape, or rape between regular or occasional partners and cohabitants, crimes committed in the name of honor, female genital and sexual mutilation, and other traditional practices harmful to women, such as forced marriages [3]. Intimate partner violence is a more specific term defining one of the most common forms of violence against women which refers to a pattern of assaultive and coercive behavior by an individual against her/his partner that may include physical injury, psychological abuse, sexual assault, progressive isolation, stalking, deprivation, intimidation, and reproductive coercion [10]. Domestic violence is now considered as a global health issue and a threat for women's health [5,10]. The worldwide prevalence of IPV among all ever partnered women was 30.0% [4]. 5.3 million cases of domestic violence among women 18 years of age and older is happening in the U.S., leading to about 2 million injuries and 1,400 deaths every year [10].

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Domestic violence is not only against women, but men have also been found to be abused by their partners [5]. One of the most common risks found in several cases of domestic violence is that the perpetrators were tested positive for drugs and/or were under the influence of alcohol, some studies found that there are strong correlations between alcohol addiction, drugs and marijuana during the act of violence [7,3,5].

Homotopy Perturbation Method (HPM) is a power series expansion method used in the solution of nonlinear partial differential equations which transforms the original non-linear differential equations into a series of linear differential equation [8,6]. [1] obtained the approximate solution of a deterministic mathematical model of zika virus using Homotopy Perturbation Method. [8] applied HPM in solving non-linear differential model equations. [6] solved a mathematical model of Ebola Virus disease transmission dynamics using HPM, the model is a coupled non-linear differential equation.[9] applied HPM to solve a system of non-linear ordinary differential equations used to model Rabies disease. The results obtained was compared with Runge-Kutta fourth order and nonstandard finite difference method and it shows to be a reliable method of solution.

Mathematical Model has been formulated to study dynamics of domestic violence [2]. In this work, a new mathematical model was formulated in cooperating violent individuals who are substance abuser.

2 Model formulation

$$\frac{dS}{dt} = \Lambda - \beta AS - \beta_1 VS - \mu S, \quad (1)$$
$$\frac{dA}{dt} = \beta AS + \beta_2 AV - (\phi_1 + \delta_1 + \varepsilon + \mu)A, \quad (2)$$
$$\frac{dV}{dt} = \beta_1 VS - (\kappa_1 + \phi_2 + \theta + \mu)V, \quad (3)$$
$$\frac{dV_a}{dt} = \beta_2 AV + \kappa_1 V - (\phi_3 + \theta + \mu)V_a, \quad (4)$$
$$\frac{dV_v}{dt} = \tau V_v + \alpha VV_v + \alpha bV_aV_v - (\phi_4 + \delta_2 + \mu)V_v, \quad (5)$$
$$\frac{dR}{dt} = \phi_1 A + \phi_2 V + \phi_3 V_a + \phi_4 V_v - \mu R, \quad (7)$$

Variables:

S = Susceptible Individuals
A = Substance Abusers
V = Violent Individuals
V_a = Violent Individuals who are substance Abusers
V_v = Victims of Domestic Violence
R = Recovered

Parameters:
β = Contact Rate
ϕ = Recovery/ Treatment rate

δ_1 = Substance abused induced death rate
δ_2 = Violence induced death rate

κ_1= movement from V to V_a
θ = Rate at which V and V_a are caught
ε = rate at which substance abusers are apprehended
τ = Victim recruitment
α = contact that results into domestic violence
b = parameter defined by increase in violence due to substance abuse

3 Method and materials

To illustrate the HPM, we consider the following non-linear differential equations given by
$K(U) - f(r) = 0, r \in \Omega,$ (8)

subject to the boundary conditions;
$B\left(U, \frac{\partial U}{\partial n}\right) = 0, r \in \Gamma,$ (9)

where K is the general differential operator, B is the boundary operator, $f(r)$ is the analytical function and Γ is the boundary of the domain Ω. The operator can be divided into two major parts L and N being the linear and non-linear component respectively, where equations (8) can be re-written as

$$L(U) + N(U) - f(r) = 0, r \in \Omega. \quad (10)$$

The HPM is structured as follows:

$$H(v, p) = (1 - p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0, \quad (11)$$

where

$$v(r, p): \Omega \in [0,1] \rightarrow R,$$

and $p \in [0,1]$ is an embedding parameter and u_0 is the initial approximation that satisfies the boundary conditions. The solution to equation (11) can be assumed as power series in p as follows:

$$V = V_0 + pV_1 + p^2V_2 + \dots \quad (12)$$

Setting $p = 1$, the approximate solution of (1), therefore can be obtained as

$$U = \lim_{p\rightarrow 1} V = V_0 + pV_1 + p^2V_2 + \dots \quad (13)$$

4 Solution of the model equation

$\frac{dS}{dt} - \Lambda + \beta AS + \beta_1 VS + \mu S = 0,$ (14)
$\frac{dA}{dt} - \beta AS - \beta_2 AV + (\phi_1 + \delta_1 + \varepsilon + \mu)A = 0,$ (15)
$\frac{dV}{dt} - \beta_1 VS + (\kappa_1 + \phi_2 + \theta + \mu)V = 0,$ (16)
$\frac{dV_a}{dt} - \beta_2 AV - \kappa_1 V + (\phi_3 + \theta + \mu)V_a = 0,$ (17)
$\frac{dV_v}{dt} - \tau V_v - \alpha VV_v - \alpha b V_a V_v + (\phi_4 + \delta_2 + \mu)V_v = 0,$ (18)
$\frac{dR}{dt} - \phi_1 A - \phi_2 V - \phi_3 V_a - \phi_4 V_v + \mu R = 0,$ (19)

with initial conditions

$$S(0) = S_0; A(0) = A_0; V(0) = V_0; V_a(0) = V_{a_0}; V_v(0) = V_{v_0}; R(0) = R_0.$$

Let

$S = a_0 + pa_1 + p^2 a_2 + \ldots$ (20)
$A = b_0 + pb_1 + p^2 b_2 + \ldots$ (21)
$V = c_0 + pc_1 + p^2 c_2 + \ldots$ (22)
$V_a = d_0 + pd_1 + p^2 d_2 + \ldots$ (23)
$V_v = e_0 + pe_1 + p^2 e_2 + \ldots$ (24)
$R = x_0 + px_1 + p^2 x_2 + \ldots$ (25)

Applying HPM to equation (14) – (19) using (20) – (25)
we have the followings:

From equation (14)

$$(1-p)\frac{dS}{dt} + p\left[\frac{dS}{dt} - \Lambda + \beta AS + \beta_1 VS + \mu S\right] = 0. \quad (26)$$

Substituting (20) – (25) into (26) and simplifying we have

$$(a_0' + pa_1' + p^2 a_2' + \cdots) + p(a_0 + pa_1 + p^2 a_2 + \cdots)(\beta(b_0 + pb_1 + p^2 b_2 + \cdots) + \beta_1(c_0 + pc_1 + p^2 c_2 + \cdots) + \mu) - \Lambda = 0. \quad (27)$$

Comparing the coefficient we get

$p^0: a_0' = 0$ (28)
$p^1: a_1' + \beta a_0 b_0 + \beta_1 a_0 c_0 + \mu a_0 = 0$ (29)
$p^2: a_2' + \beta(a_0 b_1 + a_1 b_0) + \beta_1(a_1 c_0 + a_0 c_1) = 0.$ (30)

From (15),

$$(1-p)\frac{dA}{dt}+p\left[\frac{dA}{dt}-\beta AS-\beta_2 AV+(\phi_1+\delta_1+\varepsilon+\mu)A\right]=0. \qquad (31)$$

Substituting (20) – (25) into (31) and simplifying we have

$$(b_0'+pb_1'+p^2b_2'+\cdots)+\omega_1 p(b_0+pb_1+p^2b_2+\cdots)-p(b_0+pb_1+p^2b_2+\cdots)\big(\beta(a_0+pa_1+p^2a_2+\cdots)+\beta_2(c_0+pc_1+p^2c_2+\cdots)\big)=0. \qquad (32)$$

Comparing

$$p^0: b_0'=0 \qquad (33)$$
$$p^1: b_1'+\omega_1 b_0-\beta a_0 b_0-\beta_2 b_0 c_0=0 \qquad (34)$$
$$p^2: b_2'+\omega_1 b_1-\beta(a_0b_1+a_1b_0)-\beta_2(b_1c_0+b_0c_1)=0. \qquad (35)$$

From (16),

$$(1-p)\frac{dV}{dt}+p\left[\frac{dV}{dt}-\beta_1 VS+(\kappa_1+\phi_2+\theta+\mu)V\right]=0. \qquad (36)$$

Substituting (20) – (25) into (36) and simplifying we have

$$(c_0'+pc_1'+p^2c_2'+\cdots)+p(c_0+pc_1+p^2c_2+\cdots)\big(\omega_2-\beta_1(a_0+pa_1+p^2a_2+\cdots)\big)=0 \qquad (37)$$

Comparing

$$p^0: c_0'=0, \qquad (38)$$
$$p^1: c_1'+\omega_2 c_0-\beta_1 a_0 c_0=0, \qquad (39)$$
$$p^2: c_2'+\omega_2 c_1-\beta_1(a_1c_0+a_0c_1)=0. \qquad (40)$$

From (17),

$$(1-p)\frac{dV_a}{dt}+p\left[\frac{dV_a}{dt}-\beta_2 AV-\kappa_1 V+(\phi_3+\theta+\mu)V_a\right]=0. \qquad (41)$$

Substituting (20) – (25) into (41) and simplifying we have

$$(d_0'+pd_1'+p^2d_2'+\cdots)+p\omega_3(d_0+pd_1+p^2d_2+\cdots)-p(c_0+pc_1+p^2c_2+\cdots)\big(\kappa_1+\beta_2(b_0+pb_1+p^2b_2+\cdots)\big)=0. \qquad (42)$$

Comparing

$$p^0: d_0' = 0, \quad (43)$$
$$p^1: d_1' + \omega_3 d_0 - \beta_2 b_0 c_0 - \kappa_1 c_0 = 0, \quad (44)$$
$$p^2: d_2' + \omega_3 d_1 - \beta_2 (b_1 c_0 + b_0 c_1) - \kappa_1 c_1 = 0. \quad (45)$$

From (18),

$$(1-p)\frac{dV_v}{dt} + p\left[\frac{dV_v}{dt} - \tau V_v - \alpha V V_v - \alpha b V_a V_v + (\phi_4 + \delta_2 + \mu)V_v\right] = 0. \quad (46)$$

Substituting (20) – (25) into (46) and simplifying we have

$$(e_0' + pe_1' + p^2 e_2' + \cdots) + p(e_0 + pe_1 + p^2 e_2 + \cdots)\big(\omega_4 - \tau - \alpha - \alpha b(d_0 + pd_1 + p^2 d_2 + \cdots)\big) = 0, \quad (47)$$

Comparing

$$p^0: e_0' = 0, \quad (48)$$
$$p^1: e_1' + \omega_4 e_0 - \tau e_0 - \alpha e_0 - \alpha b e_0 d_0 = 0, \quad (49)$$
$$p^2: e_2' + \omega_4 e_1 - \tau e_1 - \alpha e_1 - \alpha b(e_0 d_1 + e_1 d_0) = 0, \quad (50)$$

From (19),

$$(1-p)\frac{dR}{dt} + p\left[\frac{dR}{dt} - \phi_1 A - \phi_2 V - \phi_3 V_a - \phi_4 V_v + \mu R\right] = 0. \quad (51)$$

Substituting (20) – (25) into (51) and simplifying we have

$$(x_0' + px_1' + p^2 x_2' + \cdots) + p\mu(x_0 + px_1 + p^2 x_2 + \cdots) - p\phi_1(b_0 + pb_1 + p^2 b_2 + \cdots) - p\phi_2(c_0 + pc_1 + p^2 c_2 + \cdots) - p\phi_3(d_0 + pd_1 + p^2 d_2 + \cdots) - p\phi_4(e_0 + pe_1 + p^2 e_2 + \cdots) = 0 \quad (52)$$

Comparing

$$p^0: x_0' = 0, \quad (53)$$
$$p^1: x_1' + \mu x_0 - \phi_1 b_0 - \phi_2 c_0 - \phi_3 d_0 - \phi_4 e_0 = 0, \quad (54)$$
$$p^2: x_2' + \mu x_1 - \phi_1 b_1 - \phi_2 c_1 - \phi_3 d_1 - \phi_4 e_1 = 0. \quad (55)$$

Given that

$$\omega_1 = (\phi_1 + \delta_1 + \varepsilon + \mu); \omega_2 = (\kappa_1 + \phi_2 + \theta + \mu); \omega_3 = (\phi_3 + \theta + \mu); \omega_4 = (\phi_4 + \delta_2 + \mu). \quad (56)$$

From (28),

$$a_0' = 0. \tag{57}$$

Implying,

$$a_0 = S_0, \tag{58}$$

so that

$$b_0 = A_0, \tag{59}$$
$$c_0 = V_0, \tag{60}$$
$$d_0 = V_{a_0}, \tag{61}$$
$$e_0 = V_{v_0}, \tag{62}$$
$$x_0 = R_0. \tag{63}$$

From (29),(34),(39),(44),(49) and (54) integrating we have

$$a_1 = (-S_0(\beta A_0 + \beta_1 V_0 + \mu))t, \tag{64}$$
$$b_1 = (A_0(\beta S_0 + \beta_2 V_0 - \omega_1))t, \tag{65}$$
$$c_1 = (V_0(\beta_1 S_0 - \omega_2))t, \tag{66}$$
$$d_1 = \left(\beta_2 A_0 V_0 - \kappa_1 V_0 - \omega_3 V_{a_0}\right)t, \tag{67}$$
$$e_1 = \left(V_{v_0}(\tau + \alpha + \alpha b V_{a_0} - \omega_4)\right)t\ , \tag{68}$$
$$x_1 = \left(\phi_1 A_0 + \phi_2 V_0 + \phi_3 V_{a_0} + \phi_4 V_{v_0} - \mu R_0\right)t. \tag{69}$$

Solving (30),(35),(40),(45),(50) and (55) integrating and substituting (64) – (69) we have

$$a_2 = [-(\beta(S_0A_0(\beta S_0 + \beta_2 V_0 - \omega_1) - A_0S_0(\beta A_0 + \beta_1 V_0 + \mu)) + \beta_1(S_0V_0(\beta_1 S_0 - \omega_2) - V_0S_0(\beta A_0 + \beta_1 V_0 + \mu))]\frac{t^2}{2} \tag{70}$$

$$b_2 = [\beta(S_0A_0(\beta S_0 + \beta_2 V_0 - \omega_1) - A_0S_0(\beta A_0 + \beta_1 V_0 + \mu)) - \omega_1 A_0(\beta S_0 + \beta_2 V_0 - \omega_1) + \beta_2(V_0A_0(\beta S_0 + \beta_2 V_0 - \omega_1) + A_0V_0(\beta_1 S_0 - \omega_2))]\frac{t^2}{2} \tag{71}$$

$$c_2 = [\beta_1(S_0V_0(\beta_1 S_0 - \omega_2) - S_0V_0(\beta A_0 + \beta_1 V_0 + \mu)) - \omega_2 V_0(\beta_1 S_0 - \omega_2)]\frac{t^2}{2} \tag{72}$$

$$d_2 = \left[\beta_2(V_0A_0(\beta S_0 + \beta_2 V_0 - \omega_1) + A_0V_0(\beta_1 S_0 - \omega_2)) + \kappa_1 V_0(\beta_1 S_0 - \omega_2) - \omega_3\left(\beta_2 A_0 V_0 - \kappa_1 V_0 - \omega_3 V_{a_0}\right)\right]\frac{t^2}{2} \tag{73}$$

$$e_2 = \left[V_{v_0}(\tau + \alpha + \alpha b V_{a_0} - \omega_4)\left(\tau + \alpha + \alpha b V_{a_0}\right) + \alpha b V_{v_0}\left(\beta_2 A_0 V_0 - \kappa_1 V_0 - \omega_3 V_{a_0}\right) - \omega_4 V_{v_0}(\tau + \alpha + \alpha b V_{a_0} - \omega_4)\right]\frac{t^2}{2} \tag{74}$$

$$x_2 = \left[\phi_1 A_0(\beta S_0 + \beta_2 V_0 - \omega_1) + \phi_2 V_0(\beta_1 S_0 - \omega_2) + \phi_3\left(\beta_2 A_0 V_0 - \kappa_1 V_0 - \omega_3 V_{a_0}\right) + \phi_4 V_{v_0}(\tau + \alpha + \alpha b V_{a_0} - \omega_4) - \mu\left(\phi_1 A_0 + \phi_2 V_0 + \phi_3 V_{a_0} + \phi_4 V_{v_0} - \mu R_0\right)\right]\frac{t^2}{2}. \tag{75}$$

Substituting (58) – (75) into (20) – (25) and setting $p = 1$ we have

$$S(t) = S_0 + (-S_0(\beta A_0 + \beta_1 V_0 + \mu))t + [-(\beta(S_0 A_0(\beta S_0 + \beta_2 V_0 - \omega_1) - A_0 S_0(\beta A_0 + \beta_1 V_0 + \mu)) + \beta_1(S_0 V_0(\beta_1 S_0 - \omega_2) - V_0 S_0(\beta A_0 + \beta_1 V_0 + \mu))]\frac{t^2}{2} \tag{76}$$

$$A(t) = A_0 + (A_0(\beta S_0 + \beta_2 V_0 - \omega_1))t + [\beta(S_0 A_0(\beta S_0 + \beta_2 V_0 - \omega_1) - A_0 S_0(\beta A_0 + \beta_1 V_0 + \mu)) - \omega_1 A_0(\beta S_0 + \beta_2 V_0 - \omega_1) + \beta_2(V_0 A_0(\beta S_0 + \beta_2 V_0 - \omega_1) + A_0 V_0(\beta_1 S_0 - \omega_2))]\frac{t^2}{2} \tag{77}$$

$$V(t) = V_0 + (V_0(\beta_1 S_0 - \omega_2))t + [\beta_1(S_0 V_0(\beta_1 S_0 - \omega_2) - S_0 V_0(\beta A_0 + \beta_1 V_0 + \mu)) - \omega_2 V_0(\beta_1 S_0 - \omega_2)]\frac{t^2}{2} \tag{78}$$

$$V_a(t) = V_{a_0} + \left(\beta_2 A_0 V_0 - \kappa_1 V_0 - \omega_3 V_{a_0}\right)t + \left[\beta_2(V_0 A_0(\beta S_0 + \beta_2 V_0 - \omega_1) + A_0 V_0(\beta_1 S_0 - \omega_2)) + \kappa_1 V_0(\beta_1 S_0 - \omega_2) - \omega_3\left(\beta_2 A_0 V_0 - \kappa_1 V_0 - \omega_3 V_{a_0}\right)\right]\frac{t^2}{2} \tag{79}$$

$$V_v = V_{v_0} + \left(V_{v_0}(\tau + \alpha + \alpha b V_{a_0} - \omega_4)\right)t + \left[V_{v_0}(\tau + \alpha + \alpha b V_{a_0} - \omega_4)\left(\tau + \alpha + \alpha b V_{a_0}\right) + \alpha b V_{v_0}\left(\beta_2 A_0 V_0 - \kappa_1 V_0 - \omega_3 V_{a_0}\right) - \omega_4 V_{v_0}(\tau + \alpha + \alpha b V_{a_0} - \omega_4)\right]\frac{t^2}{2} \tag{80}$$

$$R = R_0 + \left(V_{v_0}(\tau + \alpha + \alpha b V_{a_0} - \omega_4)\right)t + \left[\phi_1 A_0(\beta S_0 + \beta_2 V_0 - \omega_1) + \phi_2 V_0(\beta_1 S_0 - \omega_2) + \phi_3\left(\beta_2 A_0 V_0 - \kappa_1 V_0 - \omega_3 V_{a_0}\right) + \phi_4 V_{v_0}(\tau + \alpha + \alpha b V_{a_0} - \omega_4) - \mu\left(\phi_1 A_0 + \phi_2 V_0 + \phi_3 V_{a_0} + \phi_4 V_{v_0} - \mu R_0\right)\right]\frac{t^2}{2} \tag{81}$$

Equation (76) – (81) are the general solutions to the model (1) – (7).

5 Conclusion

In this article a semi-analytical approach that is Homotopy Perturbation Method (HPM) was applied to obtain the general solution of a substance abuse and domestic violence mathematical model. The model equations were transformed into series form so as to convey faster. We concluded that the Homotopy perturbation method is a powerful technique to solve nonlinear differential equations.

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